Integrais Múltiplas com Maple 18

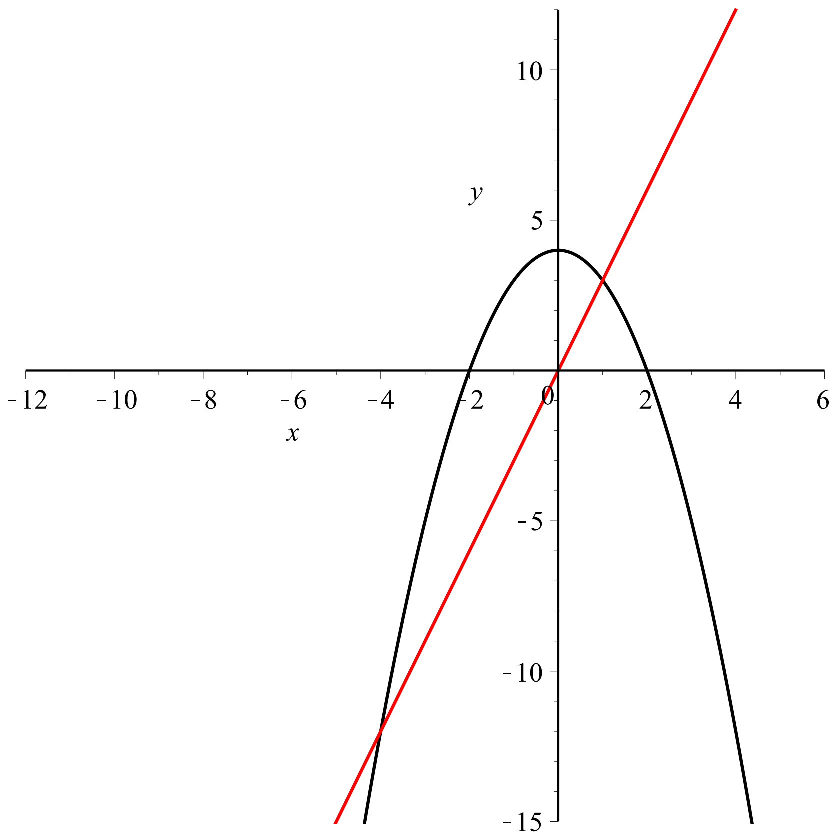
Prof. Doherty Andrade

> $with(plots)$:

Retangulares 2D

Região 2D: $\{u(x) \le y \le v(x), a \le x \le b\}$

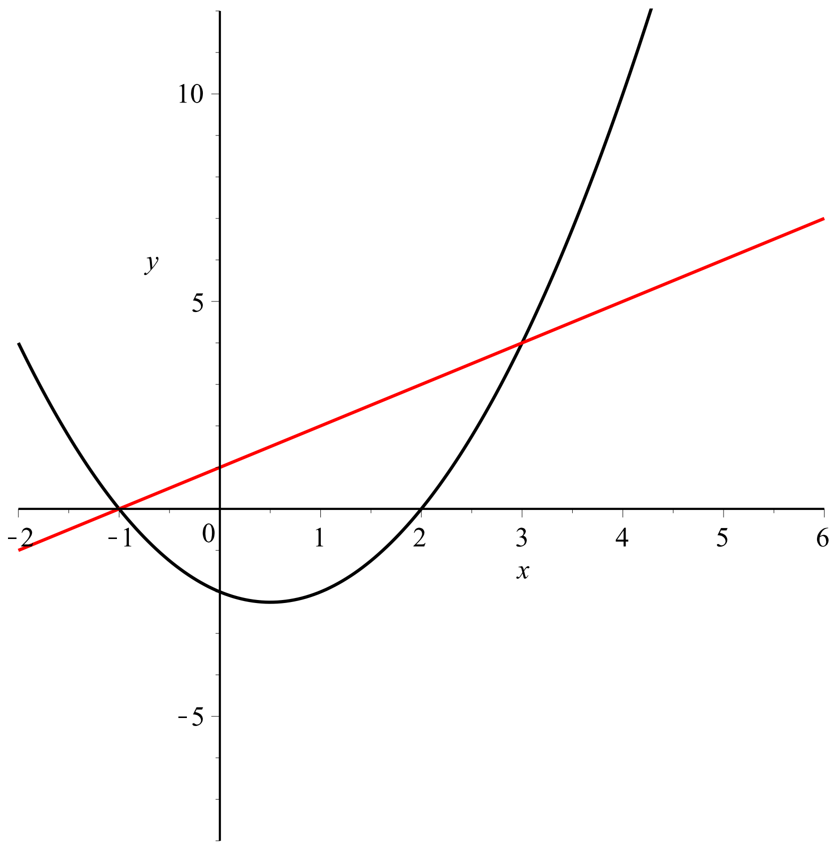
> $plot([-x^2+4, 3x], x=-12..6, y=-15..12, color=[black, red])$; *#região entre duas curvas*



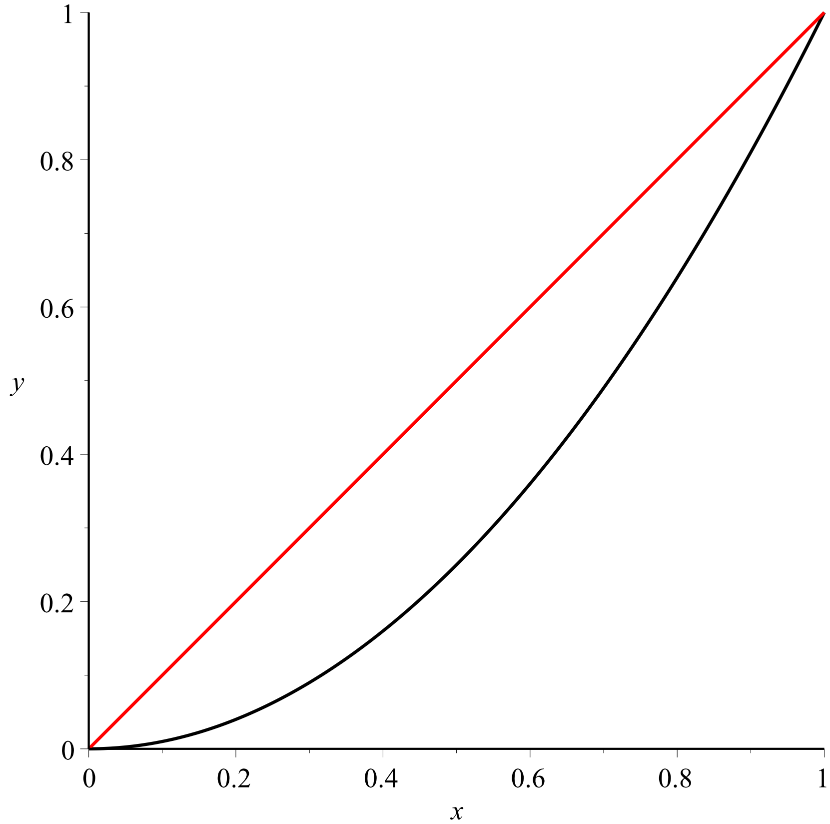
> $solve(-x^2+4=3x, x)$;

$$-4, 1 \tag{1.1}$$

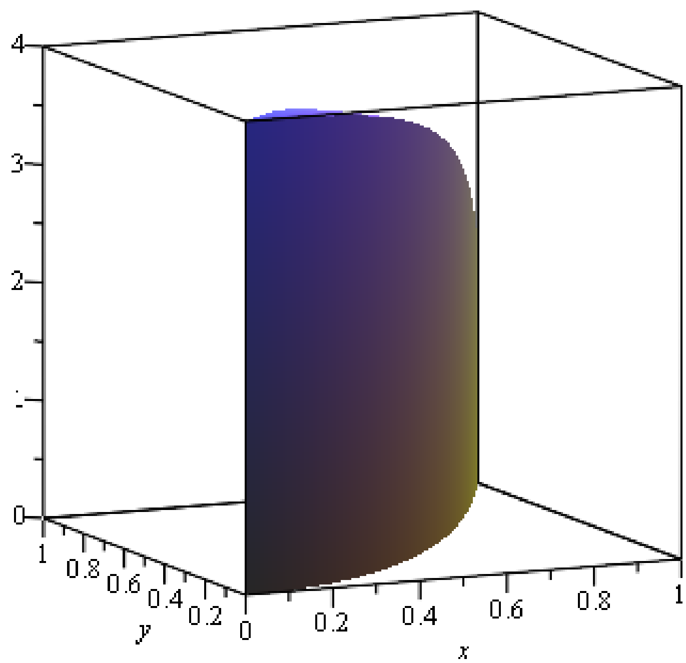
> $plot([(x+1)\cdot(x-2), 1x+1], x=-2..6, y=-8..12, color=[black, red])$;
> *#região entre duas curvas*



> $plot([x^2, x], x = 0..1, y = 0..1, color = [black, red])$; *#região entre duas curvas*

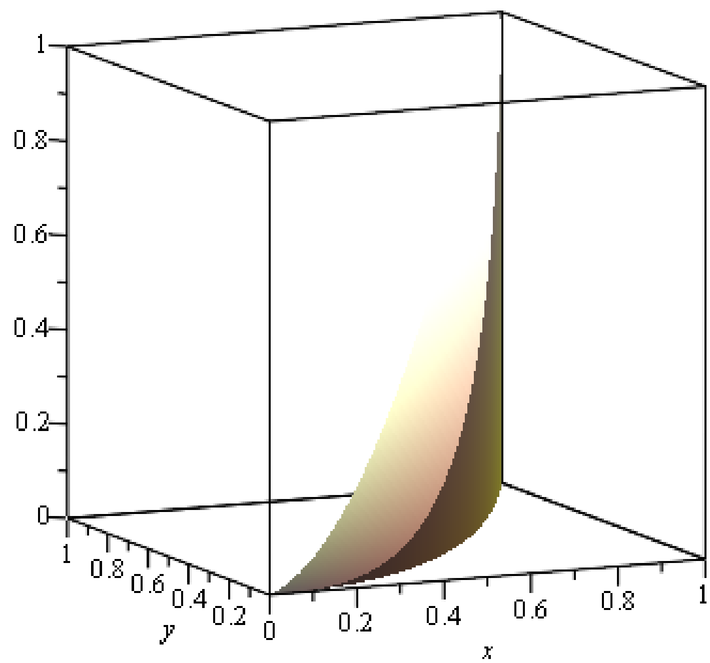


> $plot3d(4 - x^2 - y^2, x = 0..1, y = x^2..x, filled, axes = box, style = surface, orientation = [-115, 80]);$



>

> $plot3d(x\,y, x = 0..1, y = x^2..x, filled, axes = box, style = surface, orientation = [-115, 80]);$



> $Student[MultivariateCalculus][MultiInt](x \cdot y, y = x^2 .. x, x = 0 .. 1, output = steps);$

$$\int_0^1 \int_{x^2}^{x} x\, y \, \mathrm{d}y \, \mathrm{d}x$$

$$= \int_0^1 \left(\left. \frac{x\, y^2}{2} \right|_{y = x^2 .. x} \right) \mathrm{d}x$$

$$= \int_0^1 \frac{x\,(-x^4 + x^2)}{2} \, \mathrm{d}x$$

$$= \left. \left(-\frac{1}{12}\, x^6 + \frac{1}{8}\, x^4 \right) \right|_{x = 0 .. 1}$$

$$\frac{1}{24} \qquad \textbf{(1.2)}$$

>

> $Student[MultivariateCalculus][MultiInt]\left(x\sqrt{4-x^2}, y=0..2\cdot Pi, x=1..2, output=steps\right);$

$$\int_1^2\int_0^{2\pi} x\sqrt{-x^2+4}\,dy\,dx$$

$$=\int_1^2\left(x\sqrt{-x^2+4}\,y\Big|_{y=0..2\pi}\right)dx$$

$$=\int_1^2 2\,x\sqrt{-x^2+4}\,\pi\,dx$$

$$=\left.\frac{2\,(x-2)\,(x+2)\sqrt{-x^2+4}\,\pi}{3}\right|_{x=1..2}$$

$$2\sqrt{3}\,\pi \qquad \textbf{(1.3)}$$

> $Student[MultivariateCalculus][MultiInt]\left(x\cdot y\cdot z, z=1-x-y..10-x^2-y^2, y=x^2..x, x=0..1, output=steps\right)$

$$\int_0^1 \int_{x^2}^{x} \int_{1-x-y}^{-x^2-y^2+10} x\,y\,z \,dz\,dy\,dx$$

$$= \int_0^1 \int_{x^2}^{x} \left(\left. \frac{x\,y\,z^2}{2} \right|_{z=1-x-y..-x^2-y^2+10} \right) dy\,dx$$

$$= \int_0^1 \int_{x^2}^{x} \frac{x\,y\left(\left(-x^2-y^2+10\right)^2-\left(1-x-y\right)^2\right)}{2}\,dy\,dx$$

$$= \int_0^1 \left(1\,\frac{1}{2}\left(x\left(\frac{y^6}{6}+\frac{\left(2\,x^2-21\right)y^4}{4}+\frac{\left(2-2\,x\right)y^3}{3} + \frac{\left(\left(-x^2+10\right)^2-\left(1-x\right)^2\right)y^2}{2}\right)\right)\Bigg|_{y=x^2..x} \right) dx$$

$$= \int_0^1 \left(\frac{x\left(-x^{12}+x^6\right)}{12} + \frac{x\left(2\,x^2-21\right)\left(-x^8+x^4\right)}{8} + \frac{x\left(2-2\,x\right)\left(-x^6+x^3\right)}{6} + \frac{x\left(\left(-x^2+10\right)^2-\left(1-x\right)^2\right)\left(-x^4+x^2\right)}{4} \right) dx$$

$$= 1\left(-\frac{1}{168}\,x^{14}+\frac{11}{16}\,x^8-\frac{1}{48}\,x^{12}+\frac{19}{80}\,x^{10}-\frac{791}{144}\,x^6+\frac{1}{27}\,x^9+\frac{1}{6}\,x^5 - \frac{1}{14}\,x^7+\frac{99}{16}\,x^4\right)\Bigg|_{x=0..1}$$

$$\frac{26081}{15120} \qquad \textbf{(1.4)}$$

>

> $Student[MultivariateCalculus][MultiInt]\left(1+x\cdot x+y\cdot y,\, y=x^2-x-2..x+1,\, x=-2..3,\, output=steps\right)$

$$\int_{-2}^{3}\int_{x^2-x-2}^{x+1} (x^2+y^2+1)\,dy\,dx$$

$$= \int_{-2}^{3} \left(\left(\frac{1}{3}y^3 + x^2 y + y \right) \Big|_{y = x^2 - x - 2\,..\,x+1} \right) dx$$

$$= \int_{-2}^{3} \left(x^2(-x^2+2x+3) + \frac{(x+1)^3}{3} - \frac{(x^2-x-2)^3}{3} - x^2 + 2x + 3 \right) dx$$

$$= \left(-\frac{5x^4}{12} + \frac{(x+1)^4}{12} - \frac{x^7}{21} + \frac{x^6}{6} + 3x^2 + \frac{17x}{3} \right) \Big|_{x = -2\,..\,3}$$

$$\frac{800}{21} \qquad \textbf{(1.5)}$$

> *Student*[*MultivariateCalculus*][*MultiInt*]$(x \cdot y \cdot y, y = x^3 \,..\, \sqrt{x}, x = 0 \,..\, 1, output = steps)$

$$\int_{0}^{1}\int_{x^3}^{\sqrt{x}} x y^2\,dy\,dx$$

$$= \int_{0}^{1} \left(\frac{x y^3}{3} \Big|_{y = x^3\,..\,\sqrt{x}} \right) dx$$

$$= \int_{0}^{1} \frac{x(x^{3/2} - x^9)}{3}\,dx$$

$$= \left(-\frac{x^{11}}{33} + \frac{2x^{7/2}}{21} \right) \Big|_{x = 0\,..\,1}$$

$$\frac{5}{77} \qquad \textbf{(1.6)}$$

> *Student*[*MultivariateCalculus*][*MultiInt*]$\left(r\sqrt{4 - r^2}, \mathrm{t} = 0 \,..\, 2\,\mathrm{Pi}, r = 0 \,..\, 1, output = steps\right)$

$$\int_0^1\int_0^{2\pi} r\sqrt{-r^2+4}\,dt\,dr$$

$$= \int_0^1 \left(r\sqrt{-r^2+4}\,t \Big|_{t=0..2\pi} \right) dr$$

$$= \int_0^1 2\,r\sqrt{-r^2+4}\,\pi\,dr$$

$$= \left. \frac{2\,(r-2)\,(r+2)\,\sqrt{-r^2+4}\,\pi}{3} \right|_{r=0..1}$$

$$2\pi\left(-\sqrt{3}+\frac{8}{3}\right) \tag{1.7}$$

> *Student*[*MultivariateCalculus*][*MultiInt*]($r(r\cdot\cos(t)+r\cdot\sin(t)+1)$, $t=0..2$ Pi, $r=0..1$, *output* = *steps*)

$$\int_0^1\int_0^{2\pi} r(r\cos(t)+r\sin(t)+1)\,dt\,dr$$

$$= \int_0^1 \left(\int r(r\cos(t)+r\sin(t)+1)\,dt \Big|_{t=0..2\pi} \right) dr$$

$$= \int_0^1\int_0^{2\pi} r(r\cos(t)+r\sin(t)+1)\,dt\,dr$$

$$= \left. \int\int_0^{2\pi} r(r\cos(t)+r\sin(t)+1)\,dt\,dr \right|_{r=0..1}$$

$$\int_0^1\int_0^{2\pi} r(r\cos(t)+r\sin(t)+1)\,dt\,dr \tag{1.8}$$

> *Student*[*MultivariateCalculus*][*MultiInt*]($x\cdot y$, $y=x^3..3\,x+2$, $x=0..2$, *output* = *steps*)

$$\int_0^2 \int_{x^3}^{3x+2} x\,y \,dy\,dx$$

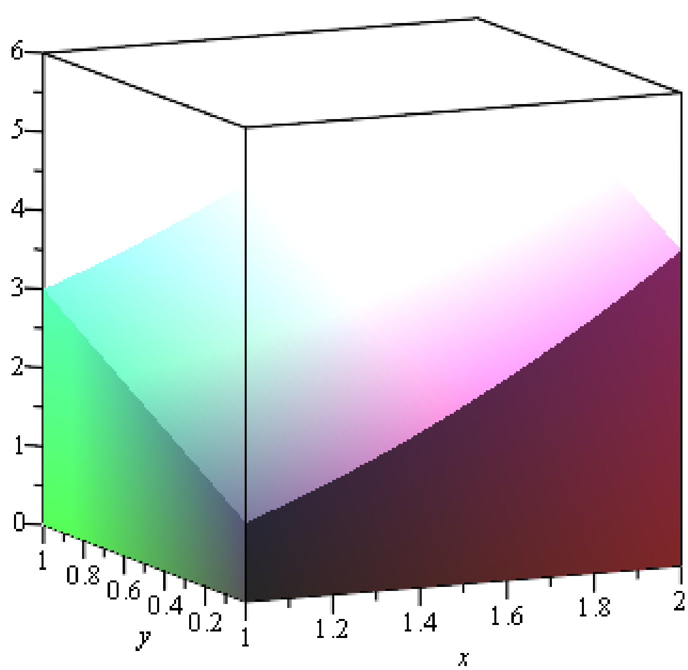
$$= \int_0^2 \left(\left. \frac{x\,y^2}{2} \right|_{y = x^3 .. 3x+2} \right) dx$$

$$= \int_0^2 \frac{x\left((3x+2)^2 - x^6\right)}{2}\,dx$$

$$= \left. \left(-\frac{1}{16}x^8 + \frac{9}{8}x^4 + 2x^3 + x^2 \right) \right|_{x = 0 .. 2}$$

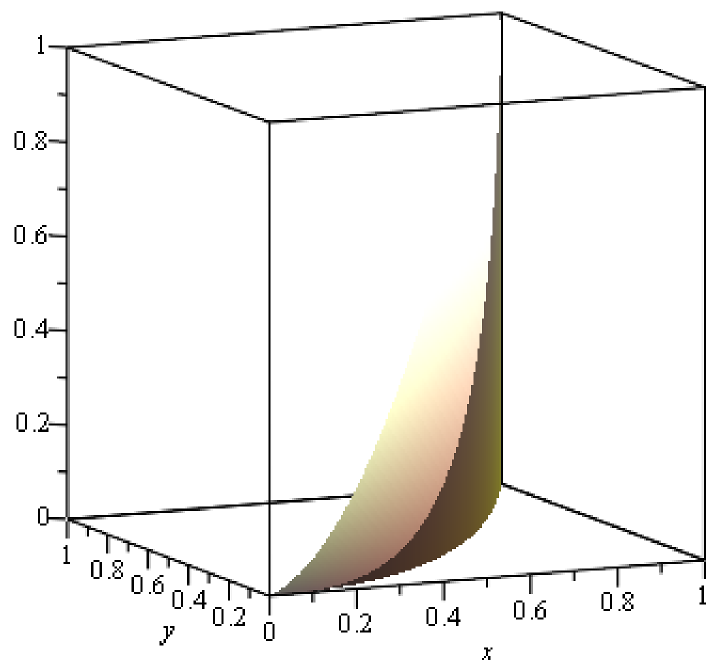
$$22 \qquad \textbf{(1.9)}$$

> $plot3d(x^2 + 2y, x = 1..2, y = 0..1, filled, axes = box, style = surface, orientation = [-115, 80]);$



>

> $plot3d(x\,y, x=0..1, y=x^2..x, filled, axes=box, style=surface, orientation=[-115, 80]);$



Visualizando Regiões de Integração em coordenadas Cartesianas em 2-D

Para trabalhar com as coordenadas cartesianas x e y, selecione o elemento de área dA, que é ou $dy\,dx$ ou $dx\,dy$, e então entre com o integrndo e os limites de integração apropriados.

Calcule o valor da integral ou exatamente ou numericamente.

Obtenha o gráfico da região plana de integração e a região 3D que está determinada pelo integrando de pelo limites de integração.

Evalie $\iint_R \Psi(x, y)\, dA$ e esboce o gráfico da região R		
Elmento de Área dA		Valor da Integral

dy dx

$\int_a^b \int_{g(x)}^{G(x)} \Psi \, dy dx$, $\Psi =$

$x \cdot y$

Clear

$\frac{1}{16}\pi - \frac{1}{8}$

Exact Numeric

$G =$ cos(x) Clear

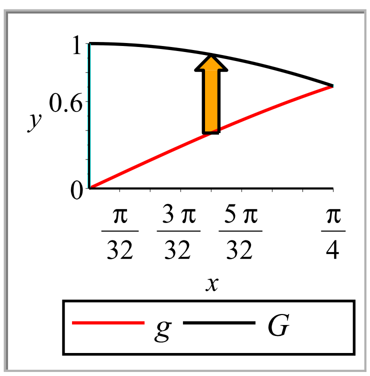
$b =$ $\frac{\pi}{4}$ Clear

$g =$ sin(x) Clear

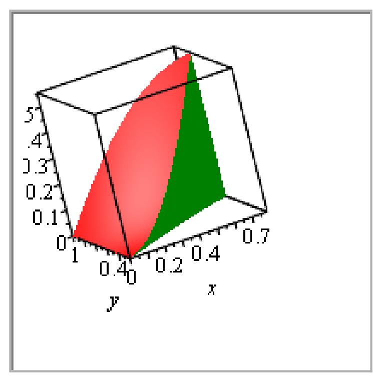
$a =$ 0 Clear

Dr... Cle... Cle...

Curvas fronteira de R



"Volume"



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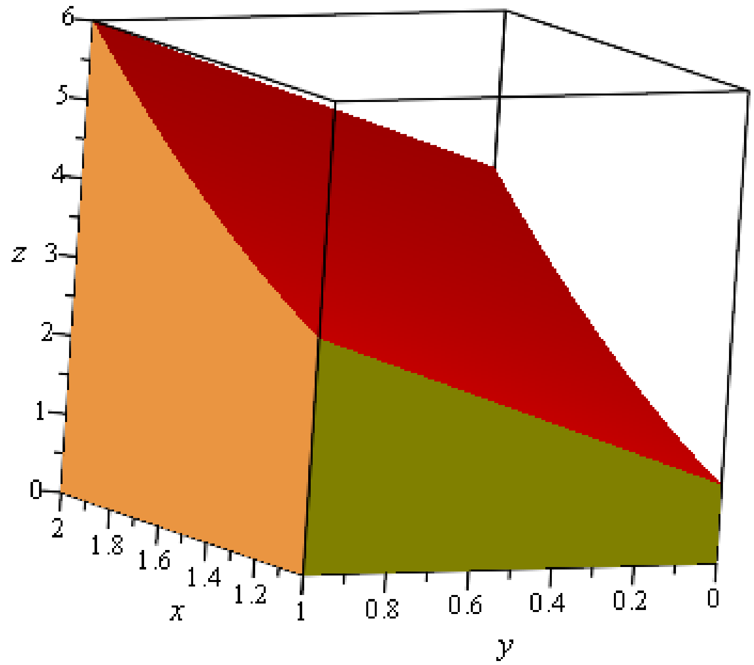
Retangulares 3D

Região 3D: $\{z_1(x, y) \le z \le z_2(x, y), y_1(x) \le y \le y_2(x), a \le x \le b\}$;

$$\{a \le x \textbf{ and } x \le b, y_1(x) \le y \textbf{ and } y \le y_2(x), z_1(x, y) \le z \textbf{ and } z \le z_2(x, y)\} \quad \textbf{(2.1)}$$

> $Student[MultivariateCalculus][MultiInt]\left(2\,x\cdot z,\ z=0\,..\,x^2+2\,y,\ y=0\,..1,\ x=1\,..2,\ output=steps\right);$



$$\int_1^2\int_0^1\int_0^{x^2+2y} 2\,x\,z\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

$$= \int_1^2\int_0^1 \left(x\,z^2\Big|_{z=0..x^2+2y} \right) \mathrm{d}y\,\mathrm{d}x$$

$$= \int_1^2\int_0^1 x\,(x^2+2\,y)^2\,\mathrm{d}y\,\mathrm{d}x$$

$$= \int_1^2 \left(\frac{x\,(x^2+2\,y)^3}{6}\Bigg|_{y=0..1} \right) \mathrm{d}x$$

$$= \int_1^2 \left(x^5 + 2\,x^3 + \frac{4}{3}\,x \right) \mathrm{d}x$$

$$= \left(\frac{1}{6}\,x^6 + \frac{1}{2}\,x^4 + \frac{2}{3}\,x^2 \right)\Bigg|_{x=1..2}$$

$$20 \qquad \textbf{(2.2)}$$

>

>

Visualizing Regions of Integration in 3-D Cartesian Coordinates

Working in Cartesian coordinates x, y, and z, select a volume element dv. (There are six possible choices: $dz\,dy\,dx$, $dz\,dx\,dy$, $dx\,dy\,dz$, $dx\,dz\,dy$, $dy\,dx\,dz$, $dy\,dz\,dx$.)

Enter an integrand and the appropriate bounds of integration.

Compute the value of the integral either exactly or numerically.

Obtain a graph of the 3-D region determined by the limits of integration. The bounding faces for the region of integration are drawn with the following color-coding: $\int_{yellow}^{gray}\int_{green}^{brown}\int_{blue}^{red} \Psi\,dv$.

Evaluate $\iiint_R \Psi(x, y, z)\,dv$ **and Graph** R	
Volume Element dv	

dz dy...

$$\int_a^b \int_{g(x)}^{G(x)} \int_{f(x,y)}^{F(x,y)} \Psi \, dz\,dy\,dx$$

, where $\Psi =$

$2 \cdot z \cdot x$

Clear

$F =$ $x^2 + 2\ \ y$

Clear

$G =$ 1

Clear

$b =$ 2

Clear

$f =$ 0

Clear

$g =$ 0

Clear

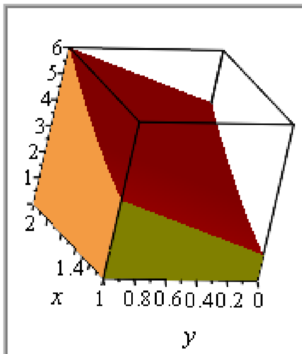
$a =$ 1

Clear

Exact Value

Floating-Point Value

20



Plot

Clear Graph

Clear All

Polares

Região: $\{r_1(\theta) \leq r \leq r_2(\theta), a \leq \theta \leq b\}$

> *Student*[*MultivariateCalculus*][*MultiInt*]((r^2), $r = 0$ .. cos(theta), $\theta = 0$ ..Pi, *coordinates* = polar[r, θ], *output* = *steps*);

$$\int_0^{\pi}\int_0^{\cos(\theta)} r^3 \, dr \, d\theta$$

$$= \int_0^{\pi} \left(\left. \frac{r^4}{4} \right|_{r = 0 .. \cos(\theta)} \right) d\theta$$

$$= \int_0^{\pi} \frac{\cos(\theta)^4}{4} \, d\theta$$

$$= \left. \left(\frac{\cos(\theta)^3 \sin(\theta)}{16} + \frac{3 \cos(\theta) \sin(\theta)}{32} + \frac{3\,\theta}{32} \right) \right|_{\theta = 0 .. \pi}$$

$$\frac{3}{32}\pi \qquad \textbf{(3.1)}$$

> *Student*[*MultivariateCalculus*][*MultiInt*](3 sqrt(r^2), $r = 0$ ..2 cos(theta), $\theta = -\frac{\text{Pi}}{2} .. \frac{\text{Pi}}{2}$, *coordinates* = polar[$r$, θ], *output* = *steps*);

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2\cos(\theta)} 3\sqrt{r^2}\, r \, \mathrm{d}r \, \mathrm{d}\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(r^2 \sqrt{r^2} \Big|_{r=0..2\cos(\theta)} \right) \mathrm{d}\theta$$

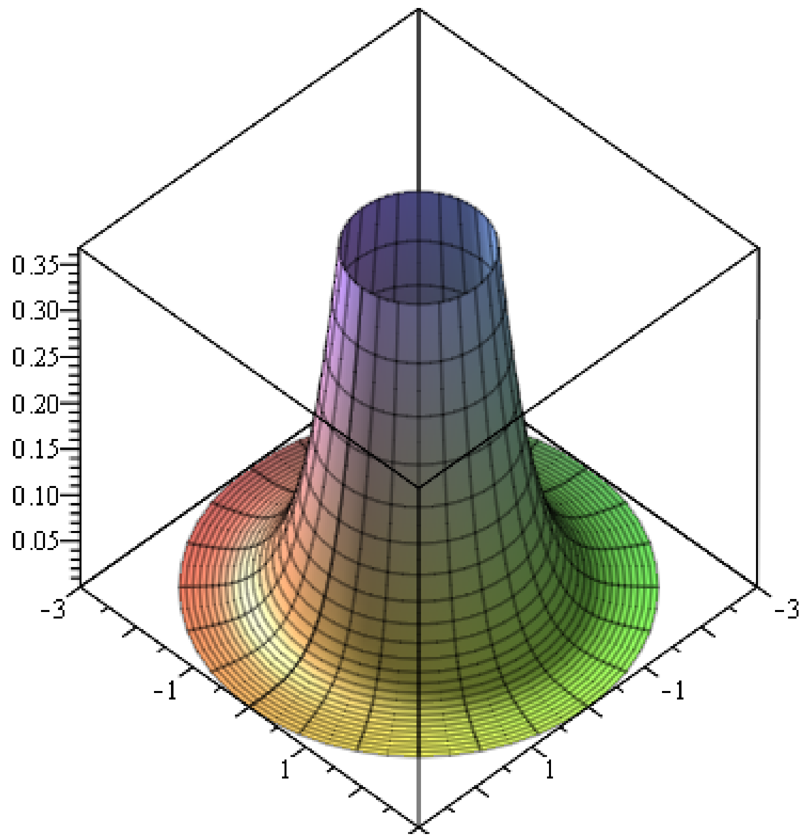
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 8\cos(\theta)^3 \operatorname{csgn}(\cos(\theta)) \, \mathrm{d}\theta$$

$$= 8 \operatorname{csgn}(\cos(\theta)) \left(\frac{\cos(\theta)^2 \sin(\theta)}{3} + \frac{2\sin(\theta)}{3} \right) \Bigg|_{\theta = -\frac{\pi}{2} .. \frac{\pi}{2}}$$

$$\frac{32}{3} \qquad \textbf{(3.2)}$$

\>

\> *Student*[*MultivariateCalculus*][*MultiInt*](exp(−r^2), r = 1 ..2, θ = 0 ..2 Pi, *coordinates* = polar[r, θ], *output* = *steps*);



$$\int_0^{2\pi}\int_1^2 e^{-r^2}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(-\frac{e^{-r^2}}{2} \Bigg|_{r=1..2} \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{e^{-1}}{2} - \frac{e^{-4}}{2} \right) d\theta$$

$$= \left(\frac{e^{-1}}{2} - \frac{e^{-4}}{2} \right) \theta \Bigg|_{\theta=0..2\pi}$$

$$e^{-1}\pi - e^{-4}\pi \quad \textbf{(3.3)}$$

Visualizing Regions of Integration in Polar Coordinates

Working in polar coordinates r and θ, select an area element dA, either $r\,dr\,d\theta$ or $r\,d\theta\,dr$, then enter an integrand and the appropriate bounds of integration.

Compute the value of the integral either exactly or numerically.

Obtain a graph of the planar region of integration and the 3-D region that is determined by the integrand and the limits of integration.

Evaluate $\iint_R \Psi(r,\theta)\,dA$ **and Graph** R

Area Element dA

$r\,dr\,d\theta$ ○

$r\,d\theta\,dr$ ○

$$\int_a^b \int_{g(\theta)}^{G(\theta)} r\,\Psi\,dr\,d\theta$$

, $\Psi =$

$e^{-(r^2)}$

Clear

Value of Integral

$$2\left(\frac{1}{2}e^{-1} - \frac{1}{2}e^{-9}\right)\pi$$

Exact | Numeric

$G =$ 3

Clear

$b =$ 2π

Clear

$g =$ 1

Clear

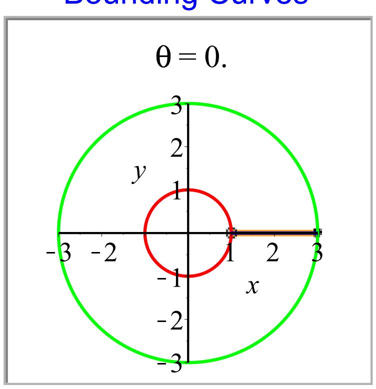
$a =$ 0

Clear

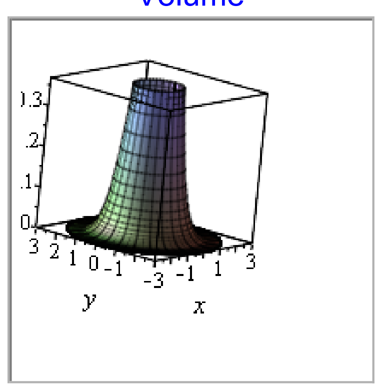
Dr...

Cle...

Bounding Curves



"Volume"



Cle...

>

Cilíndricas

Região: $\{z_1(r,\theta) \le z \le z_2(r,\theta),\, r_1(\theta) \le r \le r_2(\theta),\, a \le \theta \le b\}$

>

> Student[MultivariateCalculus][MultiInt](1, z = 0 ..8 − r cos(theta) − r sin(theta) , r = 0 ..5, θ = 0 ..2 Pi, coordinates = cylindrical[r, θ, z], output = steps);

$$\int_0^{2\pi}\int_0^{5}\int_0^{8-r\cos(\theta)-r\sin(\theta)} r \,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$$

$$= \int_0^{2\pi}\int_0^{5}\left(r\,z \Big|_{z=0..8-r\cos(\theta)-r\sin(\theta)}\right)\mathrm{d}r\,\mathrm{d}\theta$$

$$= \int_0^{2\pi}\int_0^{5} r\,(8-r\cos(\theta)-r\sin(\theta))\,\mathrm{d}r\,\mathrm{d}\theta$$

$$= \int_0^{2\pi}\left(\left(\frac{(-\cos(\theta)-\sin(\theta))\,r^3}{3}+4\,r^2\right)\Bigg|_{r=0..5}\right)\mathrm{d}\theta$$

$$= \int_0^{2\pi}\left(-\frac{125\cos(\theta)}{3}-\frac{125\sin(\theta)}{3}+100\right)\mathrm{d}\theta$$

$$= \left(-\frac{125\sin(\theta)}{3}+\frac{125\cos(\theta)}{3}+100\,\theta\right)\Bigg|_{\theta=0..2\pi}$$

$$200\,\pi \qquad (4.1)$$

> Student[MultivariateCalculus][MultiInt](z, z = r ..1, r = 0 ..1, θ = 0 ..2 Pi, coordinates = cylindrical[r, θ, z], output = steps);

$$\int_0^{2\pi}\int_0^1\int_r^1 r\,z\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(\left. \frac{r z^2}{2} \right|_{z=r..1} \right) dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \frac{r\left(-r^2+1\right)}{2}\,dr\,d\theta$$

$$= \int_0^{2\pi} \left(\left. \left(-\frac{1}{8} r^4 + \frac{1}{4} r^2 \right) \right|_{r=0..1} \right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{8}\,d\theta$$

$$= \left. \frac{\theta}{8} \right|_{\theta=0..2\pi}$$

$$\frac{1}{4}\pi \qquad \textbf{(4.2)}$$

Esféricas

Região: $\left\{\rho_1(\phi, \theta) \le \rho \le \rho_2(\phi, \theta), \phi_1(\theta) \le \phi \le \phi_2(\theta), a \le \theta \le b\right\}$

> *Student*[*MultivariateCalculus*][*MultiInt*](ρ, ρ = 0 ..1, ϕ = 0 ..Pi, θ = 0 ..2 Pi, *coordinates* = *spherical*[ρ, ϕ, θ], *output* = *steps*);

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^3 \sin(\phi)\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\left. \frac{\rho^4 \sin(\phi)}{4} \right|_{\rho = 0..1} \right) d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\sin(\phi)}{4}\, d\phi\, d\theta$$

$$= \int_0^{2\pi} \left(\left. -\frac{\cos(\phi)}{4} \right|_{\phi = 0..\pi} \right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}\, d\theta$$

$$= \left. \frac{\theta}{2} \right|_{\theta = 0..2\pi}$$

$$\pi \qquad \textbf{(5.1)}$$

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> *Student*[*MultivariateCalculus*][*MultiInt*]$\left(5\,\rho, \rho = 0..2\cos(\phi), \phi = 0..\frac{\text{Pi}}{4}, \theta = 0..2\,\text{Pi},\right.$
coordinates = *spherical*[ρ, ϕ, θ], *output* = *steps*$\left.\right)$;

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2\cos(\phi)} 5\,\rho^3 \sin(\phi)\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \left(\left. \frac{5\,\rho^4 \sin(\phi)}{4} \right|_{\rho = 0\,..\,2\cos(\phi)} \right) d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}} 20 \sin(\phi) \cos(\phi)^4\, d\phi\, d\theta$$

$$= \int_0^{2\pi} \left(\left. -4\cos(\phi)^5 \right|_{\phi = 0\,..\,\frac{\pi}{4}} \right) d\theta$$

$$= \int_0^{2\pi} \left(4 - \frac{\sqrt{2}}{2} \right) d\theta$$

$$= \left. \left(4 - \frac{\sqrt{2}}{2} \right) \theta \right|_{\theta = 0\,..\,2\pi}$$

$$8\pi - \sqrt{2}\,\pi \qquad \textbf{(5.2)}$$

> *Student*[*MultivariateCalculus*][*MultiInt*](ρ^2, $\rho = 0..1$, $\phi = 0$..Pi, $\theta = 0..2$ Pi, *coordinates* = *spherical*[ρ, ϕ, θ], *output* = *steps*);

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \rho^4 \sin(\phi)\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\left. \frac{\rho^5 \sin(\phi)}{5} \right|_{\rho = 0..1} \right) d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\sin(\phi)}{5}\, d\phi\, d\theta$$

$$= \int_0^{2\pi} \left(\left. -\frac{\cos(\phi)}{5} \right|_{\phi = 0..\pi} \right) d\theta$$

$$= \int_0^{2\pi} \frac{2}{5}\, d\theta$$

$$= \left. \frac{2\,\theta}{5} \right|_{\theta = 0..2\pi}$$

$$\frac{4}{5}\pi \qquad \textbf{(5.3)}$$

>

> *Student*[*MultivariateCalculus*][*MultiInt*]$\left(\rho, \rho = 0..1, \phi = 0..\frac{\text{Pi}}{6}, \theta = 0..2\,\text{Pi}, \textit{coordinates} = \textit{spherical}[\rho, \phi, \theta], \textit{output} = \textit{steps}\right)$;

$$\int_0^{2\pi}\int_0^{\frac{\pi}{6}}\int_0^1 \rho^3 \sin(\phi)\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{6}} \left(\left. \frac{\rho^4 \sin(\phi)}{4} \right|_{\rho = 0..1} \right) d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{6}} \frac{\sin(\phi)}{4}\, d\phi\, d\theta$$

$$= \int_0^{2\pi} \left(\left. -\frac{\cos(\phi)}{4} \right|_{\phi = 0..\frac{\pi}{6}} \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{4} - \frac{\sqrt{3}}{8} \right) d\theta$$

$$= \left. \left(\frac{1}{4} - \frac{\sqrt{3}}{8} \right) \theta \right|_{\theta = 0..2\pi}$$

$$\frac{1}{2}\pi - \frac{1}{4}\sqrt{3}\,\pi \qquad \textbf{(5.4)}$$

Visualizing Regions of Integration in Spherical Coordinates

Working in spherical coordinates ρ, ϕ, and θ, where $x = \rho \sin(\phi)\cos(\theta)$, $y = \rho \sin(\phi)\sin(\theta)$, $z = \rho \cos(\phi)$, select a volume element dv. There are six possible choices, namely, $\rho^2 \sin(\phi)$ times any one of the following:

$d\rho\ d\phi\ d\theta$,
$d\rho\ d\theta\ d\phi$,
$d\phi\ d\rho\ d\theta$,
$d\phi\ d\theta\ d\rho$,
$d\theta\ d\phi\ d\rho$,
$d\theta\ d\rho\ d\phi$

Enter an integrand and the appropriate bounds of integration.

Compute the value of the integral either exactly or numerically.

Obtain a graph of the 3-D region determined by the limits of integration. The bounding faces for the region of integration are drawn with the following color-coding: $\int_{yellow}^{gray}\int_{green}^{brown}\int_{blue}^{red} \Psi\, dv$.

Evaluate $\iiint_R \Psi(\rho, \phi, \theta)\, dv$ **and Graph** R

Volume Element $dv = \rho^2 \sin(\phi) \times$	$d\rho\, d\phi\, d\theta$	$d\rho\, d\theta\, d\phi$	$d\phi\, d\rho\, d\theta$	$d\phi\, d\theta\, d\rho$	$d\theta\, d\phi\, d\rho$	$d\theta\, d\rho\, d\phi$
	◉	○	○	○	○	○

$$\int_a^b \int_{g(\theta)}^{G(\theta)} \int_{f(\phi,\theta)}^{F(\phi,\theta)} \Psi\, \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$$

, where $\Psi =$

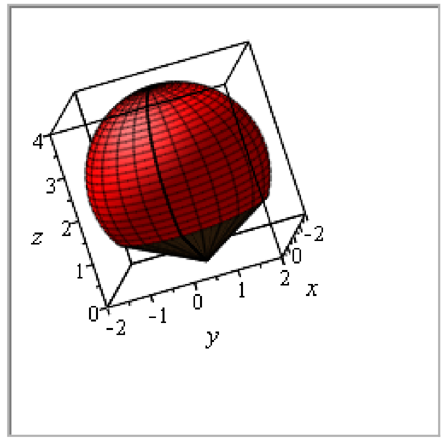
1

Clear

$F =$ $4 \cos(\phi)$ Clear	$G =$ $\frac{\pi}{3}$ Clear	$b =$ $2 \cdot \pi$ Clear
$f =$ 0 Clear	$g =$ 0 Clear	$a =$ 0 Clear

Exact Value

Floating-Point Value



$10\,\pi$

Plot

Clear Graph

Clear All

>

Área de superfície

Domain: $\{u(x) \le y \le v(x), a \le x \le b\}$

> *Student*[*MultivariateCalculus*][*SurfaceArea*]$(x \cdot y, x = 0..1, y = x^2..x, output = integral)$;

$$\int_0^1 \int_{x^2}^{x} \sqrt{x^2 + y^2 + 1}\, dy\, dx \qquad \textbf{(6.1)}$$

> *Student*[*MultivariateCalculus*][*SurfaceArea*]$(x \cdot y, x = 0..1, y = x^2..x)$; *evalf*(%);

$$\int_0^1 \left(-\frac{1}{2}\, x^2 \sqrt{x^4 + x^2 + 1} - \frac{1}{2} \ln\left(x^2 + \sqrt{x^4 + x^2 + 1}\right) x^2 - \frac{1}{2} \ln\left(x^2 + \sqrt{x^4 + x^2 + 1}\right) + \frac{1}{2}\, x \sqrt{2\, x^2 + 1} + \frac{1}{2} \ln\left(x + \sqrt{2\, x^2 + 1}\right) x^2 + \frac{1}{2} \ln\left(x + \sqrt{2\, x^2 + 1}\right) \right) dx$$

$$0.2031164292 \qquad \textbf{(6.2)}$$

> *Student*[*MultivariateCalculus*][*SurfaceArea*]$(x, x = 0..1, y = x^2..x)$; *eval*(%);

$$\frac{1}{6}\sqrt{2}$$

$$\frac{1}{6}\sqrt{2} \qquad (6.3)$$

> *Student*[*MultivariateCalculus*][*SurfaceArea*]$(x \cdot y, x = 0..1, y = x^2..x, output = integral)$;

$$\int_0^1 \int_{x^2}^{x} \sqrt{x^2 + y^2 + 1}\, dy\, dx \qquad (6.4)$$

> *Student*[*MultivariateCalculus*][*MultiInt*]$(x \cdot y, y = x^2..x, x = 0..1, output = steps)$;

$$\int_0^1 \int_{x^2}^{x} x\, y\, dy\, dx$$

$$= \int_0^1 \left(\left. \frac{x\, y^2}{2} \right|_{y = x^2..x} \right) dx$$

$$= \int_0^1 \frac{x\,(-x^4 + x^2)}{2}\, dx$$

$$= \left. \left(-\frac{1}{12} x^6 + \frac{1}{8} x^4 \right) \right|_{x = 0..1}$$

$$\frac{1}{24} \qquad (6.5)$$

> *Student*[*MultivariateCalculus*][*MultiInt*]$\left(x^2 + y^2, x = \frac{y}{2}..\sqrt{y}, y = 0..4, output = steps\right)$;

$$\int_0^4 \int_{\frac{y}{2}}^{\sqrt{y}} (x^2 + y^2)\, dx\, dy$$

$$= \int_0^4 \left(\left. \left(\frac{1}{3} x^3 + x\, y^2 \right) \right|_{x = \frac{y}{2}..\sqrt{y}} \right) dy$$

$$= \int_0^4 \left(\frac{y^{3/2}}{3} - \frac{y^3}{24} + y^2 \left(\sqrt{y} - \frac{y}{2} \right) \right) dy$$

$$= \left. \left(\frac{2\, y^{5/2}}{15} - \frac{13\, y^4}{96} + \frac{2\, y^{7/2}}{7} \right) \right|_{y = 0..4}$$

$$\frac{216}{35} \tag{6.6}$$

>