O método de Heun

Prof. Doherty Andrade

www.metodosnumericos.com.br

1 O método de Heun para PVI

Consideremos o PVI

$$\begin{cases} y' = f(x,y), a \leq x \leq b \\ y(a) = y\_0. \end{cases}$$

O método de Heun determina numericamente a solução de PVIs por meio das aproximações dadas por

$$y_{k+1} = y_k + \frac{h}{2}\left[f(x_k, y_k) + f(x_{k+1}, y_k + hf(x_k, y_k))\right], k \geq 0,$$

sendo $x_0 = a$.

O método de Heun é uma modificação do método de Euler.

O script abaixo determina numericamente a solução de PVIs utilizando o método de Heun. Veja os exemplos.

Para usar basta chamar o procedimento heun(f,x0,y0,xf,n), antes entre com os dados $f, x_0, y_0, x_f = b$ e n, o número de subintervalos.

Após a execução teremos as aproximações e o gráfico da solução.

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt

        def heun(f, x0,y0,xf,n):

            h = (xf-x0)/(n-1)
            x = np.linspace(x0,xf,n)
            y = np.zeros([n,1])

            y[0] = y0
            for i in range(1,n):
                y[i] = (h/2)*(f(x[i-1],y[i-1])+f(x[i],y[i-1]+h*f(x[i-1],y[i-1])))+y[i-1]


            print(y)
            plt.plot(x,y,'o')
            plt.xlabel("X valores")
            plt.ylabel("Y valores")
            plt.title("Solução aproximada")
            plt.show()
```

Exemplo 1: Consideremos o PVI

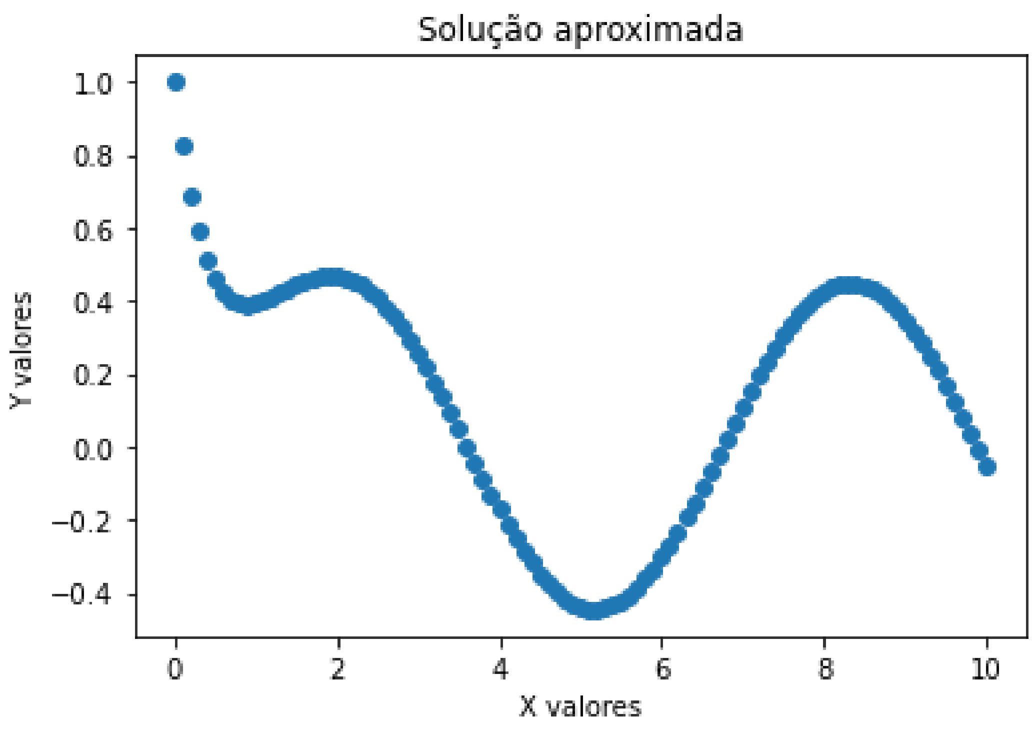
$$\begin{cases} y' = -y + \sin(x), 0 \leq x \leq 10 \\ y(0) = 1. \end{cases}$$

In [37]:
```
def f(x,y):
    return -2*y+np.sin(x)

x0 = 0
y0 = 1
xf = 10
n = 101

heun(f,x0,y0,xf,n,)
```

```
[[ 1.        ]
 [ 0.82499167]
 [ 0.69041997]
 [ 0.58886716]
 [ 0.5141628 ]
 [ 0.4611615 ]
 [ 0.42556158]
 [ 0.40375708]
 [ 0.39271732]
 [ 0.38988879]
 [ 0.39311543]
 [ 0.40057386]
 [ 0.41072082]
 [ 0.42225054]
 [ 0.43406026]
 [ 0.44522215]
 [ 0.45496064]
 [ 0.46263391]
 [ 0.46771878]
 [ 0.46979831]
 [ 0.46855149]
 [ 0.46374459]
 [ 0.45522376]
 [ 0.4429086 ]
 [ 0.42678642]
 [ 0.406907  ]
 [ 0.38337769]
 [ 0.35635876]
 [ 0.32605878]
 [ 0.29273019]
 [ 0.25666473]
 [ 0.21818891]
 [ 0.17765943]
```

```
[ 0.13545848]
[ 0.09198907]
[ 0.04767023]
[ 0.00293224]
[-0.04178819]
[-0.08605265]
[-0.1294258 ]
[-0.17147993]
[-0.2117995 ]
[-0.24998546]
[-0.2856594 ]
[-0.31846745]
[-0.3480839 ]
[-0.37421455]
[-0.39659974]
[-0.41501694]
[-0.42928311]
[-0.43925647]
[-0.44483801]
[-0.44597249]
[-0.442649  ]
[-0.4349011 ]
[-0.4228065 ]
[-0.40648627]
[-0.38610369]
[-0.36186255]
[-0.33400521]
[-0.30281012]
[-0.26858904]
[-0.23168398]
[-0.19246375]
[-0.15132026]
[-0.10866464]
[-0.06492314]
[-0.02053282]
[ 0.02406275]
[ 0.06841798]
[ 0.11208966]
[ 0.15464144]
[ 0.19564813]
[ 0.23470002]
[ 0.27140688]
[ 0.30540197]
[ 0.33634559]
[ 0.36392859]
[ 0.38787534]
[ 0.40794658]
[ 0.42394176]
```

```
[ 0.43570106]
[ 0.44310699]
[ 0.44608555]
[ 0.44460697]
[ 0.43868603]
[ 0.42838188]
[ 0.41379749]
[ 0.39507857]
[ 0.37241216]
[ 0.34602473]
[ 0.31617993]
[ 0.28317598]
[ 0.24734262]
[ 0.20903789]
[ 0.16864453]
[ 0.12656613]
[ 0.08322313]
[ 0.03904858]
[-0.00551612]
[-0.05002571]]
```



Exemplo 2: Consideremos o PVI

$$\begin{cases} y' = 4x - 2xy, 0 \leq x \leq 1 \\ y(0) = 1. \end{cases}$$

```
In [40]: def f(x,y):
             return 4*x-2*x*y # modified function to use

         x0 = 0
         y0 = 1
         xf = 1
         n = 11

         heun(f,x0,y0,xf,n,)
```

```
[[1.        ]
 [1.01      ]
 [1.039304  ]
 [1.08618596]
 [1.14795979]
 [1.22123525]
 [1.30222679]
 [1.38707601]
 [1.47214986]
 [1.55428334]
 [1.63094661]]
```